Q7

$$P = \frac{E^2 R}{(R+r)^2}$$

For maximum power $\frac{dP}{dR} = 0$, $\frac{dP}{dR}\left[\frac{E^2 R}{(R+r)^2}\right] = 0$

$$\frac{dP}{dR} = E^2 \left[R(-2)(R+r)^{-3}(1) + (R+r)^{-2}(1)\right].$$

$$= E^2 \left[\frac{-2R}{(R+r)^3} + \frac{1}{(R+r)^2}\right] = 0.$$

$$= \frac{E^2}{(R+r)^3}\left[-2R + (R+r)\right] = 0$$

$$\frac{E^2}{(R+r)^3} = 0$$

$\therefore$ Resistance; $\Rightarrow -2R + R + r = 0$

$$-R + r = 0$$

$$r = R.$$

Let's check at $R = r$

Power is maximum; take 2nd derivative

$$\frac{d^2P}{dR^2} = \frac{d}{dR}\left[\frac{-E^2 2R}{(R+r)^3} + \frac{E^2}{(R+r)^2}\right]\Bigg|_{R=r}$$

$$= \left[\frac{-2R(-3)E^2}{(R+r)^4} - \frac{2E^2}{(R+r)^3}(1) - \frac{2E^2}{(R+r)^3}\right]$$

At $R = r$,

$$= \left[\frac{6rE^2}{16r^4} - \frac{2E^2}{(R+r)^3} - \frac{2E^2}{(R+r)^3}\right]$$

$$= \left[\frac{6rE^2}{16r^4} - \frac{2E^2}{8r^3} - \frac{2E^2}{8r^3}\right] = E^2\left[\frac{3}{8r^3} - \frac{2}{8r^3} - \frac{2}{8r^3}\right].$$

$$= -\frac{1}{8r^3} E^2$$

$$= -\frac{1}{8r^3} E^2 < 0$$

$$\Rightarrow \left.\frac{d^2P}{dr^2}\right|_{R=r} < 0 \Rightarrow P \text{ is maximum at } R = r$$

$$P_{max} = \frac{E^2 r}{(r+r)^2} = \frac{E^2 r}{4r^2} = \frac{E^2}{4r}$$

$$P_{max} = \frac{E^2}{4r}$$